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MOLECULAR MIXING LENGTHS FOR TURBULENT WAKES

George W. Sutton

AVCO EVERETT RESEARCH LABORATORY

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ABSTRACT

A simplified analysis is performed of the turbulent mixing of a trace chemical species with the bulk chemical species. The general relationship between the mixing on a molecular scale and the concentration fluctuation intensity of the trace species is given. This is then applied to a turbulent wake, to determine the axial length that it takes for injected gas to become mixed. The results are sensitive to the assumed turbulent dissipation rate; giving values of the mixing length from the order of to two orders of magnitude larger than the wake radius.

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MOLECULAR MIXING LENGTHS FOR TURBULENT WAKES*

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George W. Sutton

April 1968

AVCO EVERETT RESEARCH LABORATORY
a division of
AVCO CORPORATION
Everett, Massachusetts

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Major Walter D. McComb, Jr.
SMYSE

LIST OF SYMBOLS

k_e = energy-containing wave number

m = 1 (planar wakes), 2 (axisymmetric wakes)

$\frac{1}{2}\overline{q'^2}$ = turbulent kinetic energy

R_n = drag radius, $\sqrt{C_D A/\pi}$

r = radius

S_c = Schmidt number

t = time

u = axial wake velocity

x = axial distance behind body

ϵ = rate of dissipation of turbulent kinetic energy

ϕ = turbulent mixing parameter

ξ = x/R_n

$\Delta\xi$ = non-dimensional mixing length

Subscripts and superscripts

$\overline{}$ = cross-sectional average

A, B = species

f = turbulent front

' = fluctuation

1 = point of entrainment

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Introduction

In performing calculations of chemical reactions in turbulent wakes, it is necessary to account for the rate at which the gas ingested by the turbulent front becomes mixed on a molecular scale with the gas on the interior of the turbulent wake. One proposed method uses a "mixing lag length" L, which is deduced on the basis of isotropic turbulence theory;[1] another method uses a transport equation for the electron density fluctuations to determine the mixing rate.[2] In this note, it is shown that the latter equation can be used to determine the mixing length.

Analysis

The model for determining this length is a one-dimensional wake of uniform properties of species A which ingests a small amount of species B at a distance x_1 behind the body. The flow field exterior of the

wake is also assumed to consist only of species A.

A mixing parameter ϕ is defined as follows:

$$\phi = \overline{C_A \, C_B} / \overline{C}_A \; \overline{C}_B \tag{1}$$

where C_A and C_B are the mass concentrations of species A and B respectively, and the overbar refers to a time average at a particular station in body-fixed coordinates. If the gas is unmixed, then either $C_A = 1$, $C_B = 0$, or $C_A = 0$, $C_B = 1$; so that $\overline{C_A \, C_B} = 0$; that is $\phi = 0$. If species B is intimately mixed with A, then $C_B(t)/C_A(t)$ = constant = $\overline{C}_B/\overline{C}_A$, but since then $C_A(t) = \overline{C}_A$, $\phi = 1$. Thus ϕ is a measurement of mixing on a molecular scale. Also, since there are only two species in this model

$$C_A(t) + C_B(t) = \overline{C}_A + \overline{C}_B = 1 \tag{2}$$

If we decompose each concentration into a mean value and a fluctuating component $(\;)'$, from eq. (2)

$$C_A' + C_B' = 0 \tag{3}$$

Combination of eqs. (1, 2, 3) yields:

$$\phi = 1 - \overline{C_B'^2} / \overline{C}_A \; \overline{C}_B \tag{4}$$

If B is a trace species, $\overline{C}_A \approx 1$, and eq. (4) can be rewritten as:

$$\phi(x) = 1 - \frac{\overline{C_B'^2}(x)}{\overline{C_B'^2}(x_1)} \cdot \frac{\overline{C}_B(x_1)}{\overline{C}_B(x)} \cdot \frac{\overline{C_B'^2}(x_1)}{\overline{C}_B(x_1)} \tag{5}$$

where x is the distance downstream and x_1 is the point of ingestion of species B which is initially unmixed at the point of ingestion; hence $\phi(x_1) = 0$ so that the last factor in eq. (5) is equal to unity, e.g.

$$\overline{C_B'^2}(x_1) = \overline{C}_B(x_1) \tag{6}$$

The second factor in eq. (5) is determined only by dilution. Thus the problem is reduced to that of determining the decay of mean square fluctuations of species B, for which Lin's equation (14)[2] can be used, e.g.:

$$(d/d\,\xi)\,(r_f^m\,\bar{u}\,\overline{C_B'^2}) - \overline{C}_B^2\,u_f d\,(r_f^m)/d\xi = -\,R_n\,\epsilon^{1/3}\,k_e^{2/3}\,r_f^m\,\overline{C_B'^2} \tag{7}$$

where m = 1 for plane wakes and 2 for axially symmetric wakes; $r_f(x)$ is the mean distance from the axis to the wake turbulent front; $\xi = x/R_n$; R_n is the wake drag or momentum defect radius, $\bar{u}$ is the mean wake speed, ϵ is the rate of dissipation of turbulent kinetic energy and k_e is the turbulent energy-containing wave number. The dissipation rate given in eq. (7) can be deduced from eqs. (7, 11, 20, and 23) of ref. 3, for a Schmidt S_c number of unity. This form is also applicable for $S_c < 1$, if multiplied by $\frac{1}{2}(3 - S_c^2)$.

From eq. (6), $\overline{C}_B$ is of the order of $\overline{C_B'^2}$. Since $\overline{C}_B \ll 1$, $\overline{C}_B^2 \ll \overline{C_B'^2}$ and the second term in eq. (7) can be neglected. In addition, one may approximate u_f as $\bar{u}$. Then eq. (7) can be integrated to:

$$\ln\left[r_f^m\,\overline{C_B'^2}/r_f^m(x_1)\,\overline{C_B'^2}(x_1)\right] = -\,R_n\int_{x_1}^{x}\bar{u}^{-1}\,\epsilon^{1/3}\,k_e^{2/3}\,d\xi \tag{8}$$

Now, from conservation of species B, $r_f^m \bar{C}_B \approx r_f^m (x_1) \bar{C}_B (x_1)$, and use of eq. (6) in eq. (8) yields:

$$\gamma(x) = 1 - \exp\{- R_n \int_{\xi_1}^{\xi} \bar{u}^{-1} \epsilon^{1/3} k_e^{2/3} d\xi\} \qquad (9)$$

We will next apply eq. (9) to three cases: (1) self-preserving wake; (2) the turbulence parameters calculated by Lin[2]; and (3) the turbulence parameters deduced by Schapker[4] from turbulent front measurements.

Self Preserving Wake

The rate of destruction of turbulent kinetic energy for a wake can be estimated from the generalized form of eq. (8.4) of ref. 5 for an incompressible wake:

$$\int_0^\infty \epsilon \, dr^m = - \frac{1}{2} \frac{d}{dx} \int_0^\infty \bar{u} \left[(\Delta \bar{u})^2 + \overline{q'^2} \right] dr^m \qquad (10)$$

where $\Delta \bar{u}$ is the mean velocity defect and $\frac{1}{2} \overline{q'^2}$ is the turbulent kinetic energy. For a self preserving wake, $\overline{q'^2} \sim (\Delta \bar{u})^2$. Also, from conservation of mass,

$$\Delta \bar{u} \sim U_\infty (R_n/r_f)^m \qquad (10a)$$

where U_∞ is the free stream velocity. Thus,

$$\epsilon(x) \sim U_\infty^3 (R_n/r_f)^{3m} x^{-1} \qquad (11)$$

Since the self preserving wake is also geometrically similar, the energy-containing wave number is inversely proportional to the wake radius,

given by

$$(r_f/R_n) \sim \xi^{\frac{1}{m+1}} \tag{12}$$

and the mean wake velocity is taken as approximately U_∞. Use of eqs. (11, 12) in (9) yields:

$$\phi = 1 - (\xi_1/\xi)^c \tag{13}$$

where c is the constant representing the numerical constants of proportionality in eqs. (11, 12) and between k_e and $1/r_f$. The important conclusion from (13) is that the distance required to achieve a given amount of mixing depends linearly on ξ_1, the distance at which the species was initially ingested: that is, the further downstream the ingestion, the longer the length to become well-mixed. From eq. (13) the length $\Delta\xi = \xi - \xi_1$, to achieve a certain degree of mixing is

$$\Delta\xi/\xi_1 = (1-\phi)^{-1/c} - 1 \tag{14}$$

From eq. (14), $\Delta\xi \to \infty$ as $\phi \to 1$ but for finite values of ϕ and $c^{-1}\ell n(1-\phi) << 1$

$$\Delta\xi \approx -c^{-1}\xi_1 \ell n(1-\phi) \tag{15}$$

In general, the integrand of eq. (9) can be represented as:

$$R_n \bar{u}^{-1} \epsilon^{1/3} k_e^{2/3} = c\,\xi^{-n} \tag{16}$$

and eq. (9) can be integrated as follows:

$$\phi = 1 - \exp\left[- c\,(n-1)^{-1}\,(\xi_1^{\,1-n} - \xi^{\,1-n})\right] \tag{17}$$

For either c large or ϕ small, eq. (17) becomes:

$$\Delta\xi \approx - c^{-1} \left(\xi_1\right)^n \ell n\,(1-\phi) \tag{18}$$

which is the generalization of eq. (15). In order for the mixing length $\Delta\xi$ to be a constant multiple of the distance from the axis to the turbulent front from eq. (12) $n = (m+1)^{-1}$; e.g. 1/2 for plane wakes and 1/3 for axially symmetric wakes. This condition is almost fulfilled for Schapker's data[4] for $\xi < 6 \times 10^3$, where n = 0.43, as shown below.

Equation (18) can also be written as

$$\Delta\xi = - \ell n\,(1-\phi)\left[R_n\,\bar{u}^{-1}\,\epsilon^{1/3}\,k_e^{\,2/3}\right]^{-1}_{\xi_1} \tag{19}$$

which could have been obtained directly from eq. (9) by taking the integrand constant at its value at ξ_1. If one defines a characteristic decay time τ as

$$\tau \approx \epsilon^{-1/3}\,k_e^{\,-2/3} \tag{20}$$

and a non-dimensional decay length $\Lambda = U_\infty\,\tau/R_n$, then eq. (19) becomes

$$\Delta\xi = - \Lambda\,\ell n\,(1-\phi) \tag{21}$$

which yields the expected exponential decay of ϕ. The use of a definite mixing lag is obviously somewhat arbitrary, since $\Delta\xi$ depends on the criteria for ϕ. To compare various theories, an arbitrary value of ϕ equal to 1/2 has been chosen.

Hypersonic Turbulence Parameters

Lin[2] assumed $k_e \approx r_n^{-1}$ and also calculated ϵ. A good match to those results for $\xi > 10^2$ is $c = 0.465$, $n = 2/3$, so that for $\phi = \frac{1}{2}$

$$\Delta\xi \approx 1.49\ \xi_1^{2/3} \tag{22}$$

which is shown in Fig. 1 in comparison to the wake radius. It is seen that the mixing is between one and two orders of magnitude larger than the mean radius of the turbulent front, which is primarily a consequence of the low values of ϵ. Schapker's results[4] can be represented as follows

$$\begin{aligned} R_N\, \bar{u}_{.}^{-1}\, \epsilon^{1/3}\, k_e^{2/3} &= 86.5\, \beta^{5/6}\, (T_f/T_\infty)^{0.4}\, Re_{d_\infty}^{-0.25}\, \xi^{-0.43}\ ;\ \xi \leq 2000 \\ &= 6.75 \times 10^3\, (T_f/T_\infty)^{0.4}\, \beta^{5/6}\, Re_{d_\infty}^{-0.25}\, \xi^{-1};\ \xi > 2000 \end{aligned} \tag{23}$$

where $\beta = u_f/U_\infty \approx 1$ and T is temperature. Neglecting the factor T_f/T_∞, the mixing length for $\phi = \frac{1}{2}$ is then given by:

$$\begin{aligned} \Delta\xi &= 1.15 \times 10^{-2}\, (\ell n\ 2)\, Re_{d_\infty}^{0.25}\, \xi_1^{0.43} & \xi_1 \leq 2 \times 10^3 \\ &= 1.48 \times 10^{-4}\, (\ell n\ 2)\, Re_{d_\infty}^{0.25}\, \xi_1 & \xi_1 > 2 \times 10^3 \end{aligned} \tag{24}$$

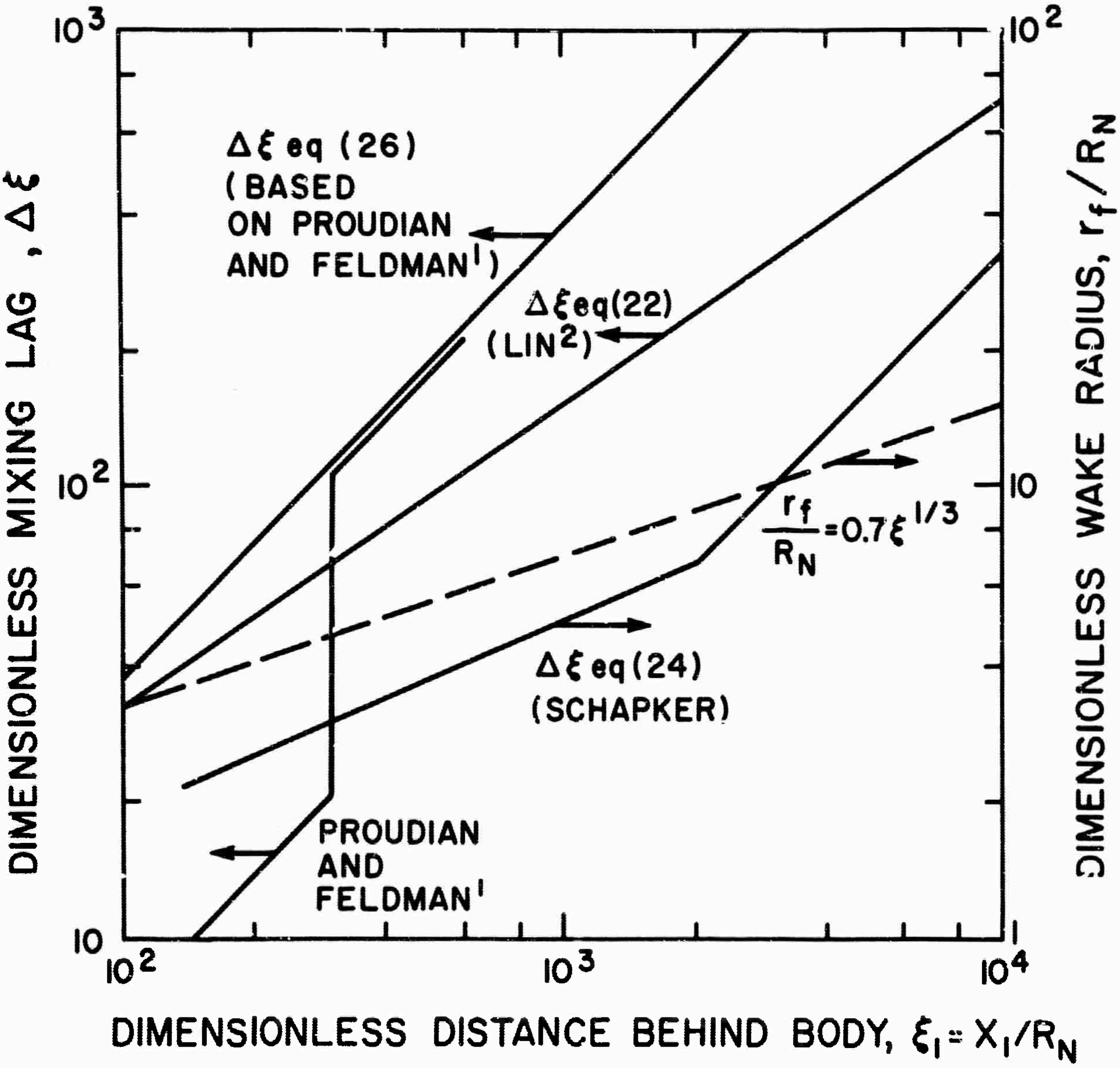

Fig. 1 Present mixing length calculations based on Lin's[2] and Schapker's[4] turbulence parameters for $R_{e_{d_\infty}} = 10^6$, compared with the wake radius and previous estimates.[1]

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and is also shown in Fig. 1. These values result in a short mixing length, approximately equal to the wake radius within about a factor of 2. These short mixing lengths are caused primarily by the large values of both ϵ and k_e deduced by Schapker.[4]

Proudian and Feldman's analysis[1] results in:

$$\Lambda = \xi_1 \left[1 - \exp - (\rho/\rho_\infty)^{2/3}\, n^{-1}\right] \tag{25}$$

where ρ is mass density and $n^{\frac{1}{2}}$ is the exponent of the defect velocity decay. For a self similar wake $n = 4/3$, and for $\rho = \rho_\infty$, $\phi = \frac{1}{2}$,

$$\Delta\xi = 0.37\ \xi_1 \tag{26}$$

also shown in Fig. 1. Here it is seen that this mixing length is much larger than any of the others, with a considerably larger slope. The values recommended in ref. 1 are also shown in Fig. 1, adjusted for $\phi = \frac{1}{2}$, which has an even larger average slope. (For $\xi > 600$, a constant mixing time of 2 msec is recommended.)

Discussion

The mixing length, is deduced from various theoretical calculations and measurements are obviously in wide disagreement by as much as two orders of magnitude. Unfortunately, direct measurements are not available, but calculations have been performed of the effect of various mixing lengths on electron densities in wakes.[6] The calculations shown in Fig. 10 of ref. 6 indicate that the results are insensitive to the mixing length for $\xi < 2500$, but becomes very sensitive beyond that.

Furthermore, Figs. 3 and 10 of ref. 6 which compares the calculations with experiments[7] indicate that the mixing length is quite short, e. g. $\Delta \xi / r_f \leq 10$ for $x/R_N < 4000$. These valves are less than those predicted by Lin by only a factor of about 3 at $x/R_N = 4000$. The small values predicted by Schapker may be implausible. Additional experimental evidence would be desirable to resolve these differences.

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